

The statistical properties of the city transport in Cuernavaca (Mexico) and Random matrix ensembles

Milan Krbálek^{1,3} and Petr Šeba^{2,3}

¹ *Department of Mathematics, Faculty of Nuclear Sciences and Physical Engineering, Trojanova 13, Prague Czech republic*

² *Institute of Physics, Czech Academy of Sciences, Cukrovarnicka 10, Prague, Czech Republic*

³ *Department of Physics, Pedagogical University, Vita Nejedleho 573, Hradec Kralove, Czech Republic*

We analyze statistical properties of the city bus transport in Cuernavaca (Mexico) and show that the bus arrivals display probability distributions conforming those given by the Unitary Ensemble of random matrices.

It is well known that the statistical properties of coherent chaotic quantum systems are well described by the Wigner/ Dyson random matrix ensembles. The fact that the spectral statistics of such chaotic systems is to a large extent generic - a phenomenon known as the universality of quantum chaos - has been confirmed both theoretically and experimentally. (See for instance [1] for references.)

The statistical distributions characterizing the ensembles of random matrices can be understood as minimizing the information contained in the system with the constraints that the matrices possess some discrete symmetry properties [2], [3]. Let $P(x_1, x_2, \dots, x_n)$ denotes the joint probability distribution of the eigenvalues x_1, x_2, \dots, x_n of the given matrices and

$$I = - \int P(x_1, x_2, \dots, x_n) \ln(P(x_1, x_2, \dots, x_n)) dx_1 \dots dx_n \quad (1)$$

be its information content. Assuming for instance that the matrices are invariant with respect to a time reversal transformation the information I is minimized when the distribution $P(x_1, x_2, \dots, x_n)$ describes Orthogonal ensemble (GOE). If there is not external symmetry the total minimum of the information I is achieved for the Unitary ensemble (GUE), where the only constraint is that the matrices should be hermitean.

It is known for a long time that matrix ensembles are of relevance also for classical one dimensional interacting many particle systems, where the matrix eigenvalues x_1, x_2, \dots, x_n describe the positions of the particles. So the thermal equilibrium of a one-dimensional gas interacting via Coulomb potential (Dyson gas) has statistical properties (depending on temperature) that are identical with those of random matrix ensembles [4]. The same holds true also for other potentials. An example is the Pechukas gas [5] where the one dimensional particles interact by a potential $\lambda V(x)$ with $V(x) = 1/|x|^2$, x being their mutual distance and λ the relevant coupling constant. Regarding the couplings λ as additive canonical variables it has been shown by Pechukas [5] and Yukawa [6], that the statistical equilibrium of the related canonical ensemble is described by random matrix theory. It

has to be stressed however, that those results were obtained under special requirements on the dynamics of the variables λ , ensuring in fact full equivalence of the system to matrix diagonalization. Nevertheless the methods of statistical physics remain valid also for different shapes of the particle potential as well as for different dynamics of the coupling variables λ . It can be shown that the potential

$$V(x) \approx 1/|x|^a \quad (2)$$

with a being positive constant, leads also to random matrix distribution of the particle positions. The equivalence of the statistical properties of the particle positions of one dimensional interacting gases to random matrix ensembles and the fact that GUE minimizes the information (1) lead us to speculate, that whenever the information contained in the gas is minimized its properties are described by GUE. However, according to our best knowledge, this fact was never tested.

The one dimensional gas to be studied in the present letter is represented by buses that operate the city line number 4 in Cuernavaca (Mexico). We will show that the statistical properties of the bus arrivals are described by the Unitary Ensemble of random matrices. To explain the origin of the interaction between subsequent buses several remarks are necessary. First of all it has to be stressed that there is not a covering company responsible for organizing the city transport. Consequently such constraints like a time table etc. that represent external influence on the transport do not exist. Moreover, each bus is a property of the driver. The drivers try to maximize their income and hence the amount of passengers they transport. This leads to competition among the drivers and to their mutual interaction. It is clear that without interaction the probability distribution of the distances between subsequent buses will be Poissonian. (This is due to the rather complicated traffic conditions in the city that work as an effective randomizer). Poisson distribution implies, however, that the probability of close encounters of two buses is high (bus clustering) which is in conflict with the effort of the driver to maximize the number of transported passengers and accordingly

maximize the distance to the preceding bus. In order to avoid the unpleasant clustering effect the bus drivers in Cuernavaca engage people that record the arrival times of buses on significant places. Arriving at a checkpoint, the driver gets the information when the previous bus passed that place. Knowing the time interval to the preceding bus the driver tries to optimize the distance to it either by slowing down or speeding up. In such a way the obtained information leads to interaction between buses and changes their statistical properties.

We have collected records catching the arrivals of the buses of the line No.4 close to the city center. The record contains altogether 3500 arrivals during a time period of 27 days whereby the arrivals on different days are regarded as statistically independent. After unfolding the peak times we evaluated the related probability distributions and compared them with the predictions of GUE. In particular we have focused on the bus spacing distribution, i.e. on the probability density $P(s)$ that spacing between two subsequent buses equals to s and on the bus number variance $N(T)$ measuring the fluctuations of the total number $n(T)$ of buses arriving to the place during the time interval T :

$$N(T) = \langle (n(T) - T)^2 \rangle \quad (3)$$

where $\langle \rangle$ means the sample averaging. (Note that after unfolding the mean distance between buses equals to 1.) According to the prediction of the unitary ensemble the spacing distribution and the number variance are given by

$$P(s) = \frac{32}{\pi^2} s^2 \exp\left(-\frac{4}{\pi} s^2\right) \quad (4)$$

and

$$N(T) \approx \frac{1}{\pi^2} (\ln 2\pi T + \gamma + 1) \quad (5)$$

Those predictions are compared with the obtained bus arrival data and displayed on the following figures:

Figure 1 shows the bus interval distribution when compared with the GUE prediction (4). The bus data are marked by (+). The minor discrepancy between the GUE prediction and the bus data can be explained taking into account the fact that the bus data do not represent the full record. Assuming that roughly 0.8% of the bus arrivals is not notified and rejecting the same amount of randomly chosen data from the random matrix eigenvalues, we get very satisfactory agreement. Due to the limited amount of records available the bus interval distribution is sensitive to the binning used in the evaluation

of the probability density $P(s)$.

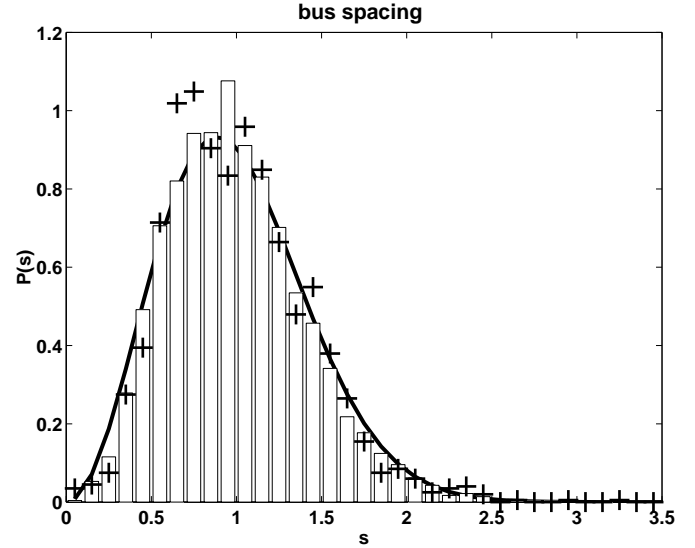


FIG.1. Bus interval distribution $P(s)$ obtained for the city line No. 4. The full line represents the random matrix prediction (4), the markers (+) represent the bus interval data and bars display the random matrix prediction (4) with 0.8% of the data rejected

This is why on the next figure we plot the integrated interval distribution $I(s) = \int P(s') ds'$ that is not liable to binning fluctuations. The agreement with the GUE distribution is evident.

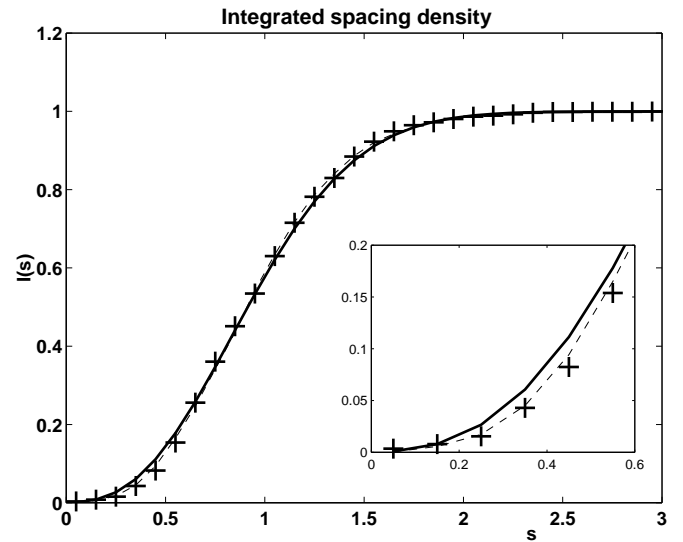


FIG.2. The integrated bus interval distribution $I(s)$ The full line represents the random matrix prediction, the markers (+) represent the bus data and dashed line shows the random matrix prediction with 0.8% of the data rejected. The distribution close to the origin is magnified on the insert

The next figure shows the number variance (5) obtained for GUE and compared with the bus data. Here the agreement is good up to time interval $T \approx 3$. For larger T the number variance of the bus arrivals lies significantly above the prediction given by (5). This in-

indicates that the long range correlations between more than three buses are weaker than predicted by the Unitary ensemble. The explanation is simple: getting the time interval information of the preceding bus the driver tries to optimize his position. Doing so he has, however, to take into account also the assumed interval to the bus behind him since otherwise this bus will overtake him. Hence the driver tries to optimize his position between the preceding and following bus that leads to the observed correlation.

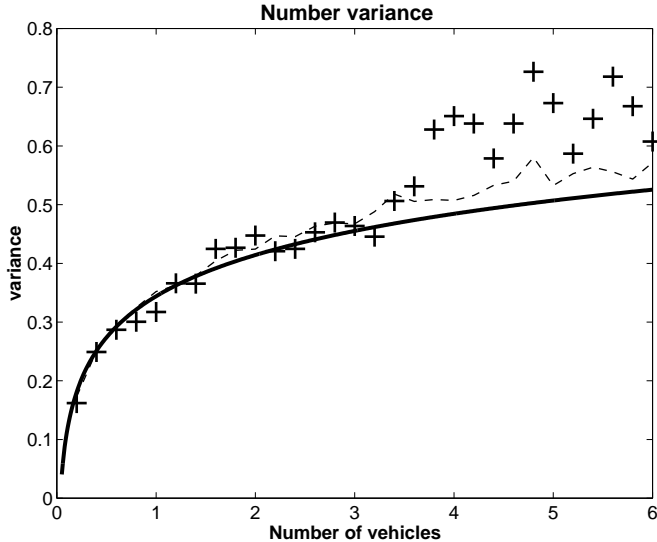


FIG.3. The number variance $N(s)$. Full line represents the random matrix prediction (5), the markers (+) represent the bus data and the dashed line display result obtained with the help of the potential (6) with the summation restricted to three neighboring particles only.

The GUE properties of the bus arrival statistics can be understood when regarding the buses as one dimensional interacting gas. It was already mentioned that the exact GUE statistics is obtained for Coulomb interaction between the gas particles, i.e. for the interaction potential V given by

$$V = - \sum_{i < j} \log(|x_i - x_j|) + \frac{1}{2} \sum_i x_i^2 \quad (6)$$

(In (6) the second terms represents a force confining the gas close to origin and is not important for our discussion. Equivalently one can discuss a one dimensional gas on a circle instead and then the second term is missing). The statistical properties of the particle positions of the Dyson gas are identical with those of the random matrix ensembles [7], [8]. In particular the properties of the unitary ensemble are recovered by minimizing the information contained in the particle positions.

It is of interest that similar potential can be indeed found when studying the reaction of driver on the traffic situation. Here we can use older results describing the behaviour of highway drivers. For one dimensional models it was shown [9] that the i -th driver accelerate

according to

$$\frac{dv_i}{dt} \approx \frac{f(v_{i+1}, v_i)}{x_{i+1} - x_i} \quad (7)$$

where x_{i+1} and v_{i+1} represent the position and velocity of the preceding car respectively and $f(v_{i+1}, v_i)$ is a function depending on the car velocities only. Approximating f by a constant (justified for low velocities) we get that the cars accelerate in the same way as described by the Coulomb interaction (6).

The exact form of the potential is, however, not crucial for the result. Using Metropolis algorithm [8] we have numerically evaluated the equilibrium distributions of the positions of one dimensional gas interacting via potential (2). When the exponent a is fixed and $a < 2$ the resulting equilibrium distributions belong to the same class as in the Dyson case (6). The numerical results show clearly that for a given a one can always find such a temperature of the gas that the equilibrium distribution is given by GUE. Moreover, the fact that the original Dyson potential (6) contains interaction between all pairs of the gas particles is also not substantial. Numerical simulations show that a good agreement with the random matrix theory is obtained when the summation in (6) is restricted and involves three neighboring particles only.

The exact interaction between buses in Cuernavaca is not known. However the weak sensitivity of the statistical equilibrium to the exact form of the potential guide us to the conviction that unitary ensembles are a good choice for bus description.

We conclude that the statistical properties of the city bus transport in Cuernavaca can be described by Gaussian Unitary Ensemble of random matrices. This behavior can be understood as equilibrium state of interacting one dimensional gas under the assumption that the information contained in the positions of individual gas particles is minimized. The agreement of the actual bus data with the GUE prediction is surprisingly good.

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